CollegeBoard
Advanced Placement
Program

# AP® Calculus BC
# 2004 Free-Response Questions
# Form B

**CALCULUS BC**
**SECTION II, Part A**
**Time—45 minutes**
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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1. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time $t$, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of $t$. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.

(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.

(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

(d) Find the $x$-coordinate of the position of the particle at time $t = 3$.

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2. Let $f$ be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for $f$ about $x = 2$ is given by

$$T(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3.$$

(a) Find $f(2)$ and $f''(2)$.

(b) Is there enough information given to determine whether $f$ has a critical point at $x = 2$ ?

If not, explain why not.

If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether $f$ has a critical point at $x = 0$ ?

If not, explain why not.

If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(d) The fourth derivative of $f$ satisfies the inequality $\left|f^{(4)}(x)\right| \le 6$ for all $x$ in the closed interval $[0, 2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.

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| $t$ (minutes) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| $v(t)$ (miles per minute) | 7.0 | 9.2 | 9.5 | 7.0 | 4.5 | 2.4 | 2.4 | 4.3 | 7.3 |

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time $t$ minutes, where $v$ is a differentiable function of $t$. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function $f$, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

(d) According to the model $f$, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

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**END OF PART A OF SECTION II**

**CALCULUS BC**

**SECTION II, Part B**

**Time—45 minutes**

**Number of problems—3**

**No calculator is allowed for these problems.**

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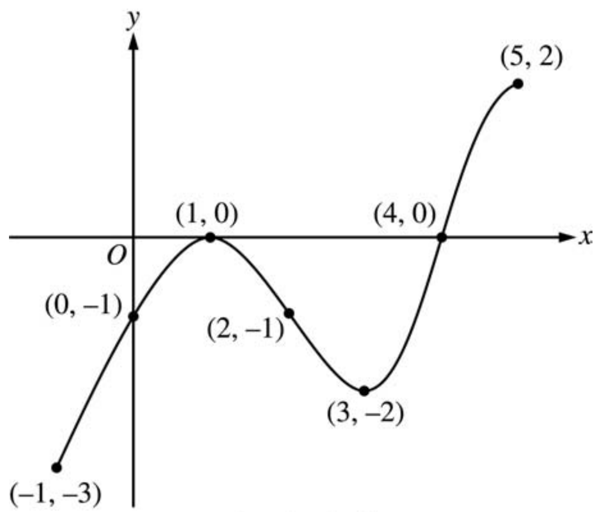


Graph of $f'$

4. The figure above shows the graph of $f'$, the derivative of the function $f$, on the closed interval $-1 \le x \le 5$. The graph of $f'$ has horizontal tangent lines at $x = 1$ and $x = 3$. The function $f$ is twice differentiable with $f(2) = 6$.

   (a) Find the $x$-coordinate of each of the points of inflection of the graph of $f$. Give a reason for your answer.

   (b) At what value of $x$ does $f$ attain its absolute minimum value on the closed interval $-1 \le x \le 5$ ? At what value of $x$ does $f$ attain its absolute maximum value on the closed interval $-1 \le x \le 5$ ? Show the analysis that leads to your answers.

   (c) Let $g$ be the function defined by $g(x) = x f(x)$. Find an equation for the line tangent to the graph of $g$ at $x = 2$.

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5. Let $g$ be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

   (a) Find the average value of $g$ on the closed interval $[1, 4]$.

   (b) Let $S$ be the solid generated when the region bounded by the graph of $y = g(x)$, the vertical lines $x = 1$ and $x = 4$, and the $x$-axis is revolved about the $x$-axis. Find the volume of $S$.

   (c) For the solid $S$, given in part (b), find the average value of the areas of the cross sections perpendicular to the $x$-axis.

   (d) The average value of a function $f$ on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b\to\infty}\left[\frac{\int_a^b f(x)\,dx}{b-a}\right]$. Show that the improper integral $\int_4^\infty g(x)\,dx$ is divergent, but the average value of $g$ on the interval $[4, \infty)$ is finite.

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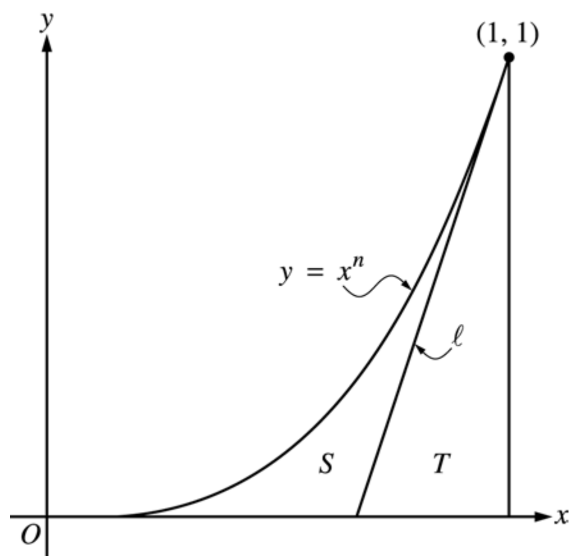


6. Let $\ell$ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

   (a) Find $\int_0^1 x^n\,dx$ in terms of $n$.

   (b) Let $T$ be the triangular region bounded by $\ell$, the $x$-axis, and the line $x = 1$. Show that the area of $T$ is $\frac{1}{2n}$.

   (c) Let $S$ be the region bounded by the graph of $y = x^n$, the line $\ell$, and the $x$-axis. Express the area of $S$ in terms of $n$ and determine the value of $n$ that maximizes the area of $S$.

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**END OF EXAMINATION**