



**AP<sup>®</sup> Calculus BC  
2004 Free-Response Questions  
Form B**

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# 2004 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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1. A particle moving along a curve in the plane has position  $(x(t), y(t))$  at time  $t$ , where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \quad \text{and} \quad \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of  $t$ . At time  $t = 0$ , the particle is at the point  $(4, 1)$ .

- (a) Find the speed of the particle and its acceleration vector at time  $t = 0$ .
  - (b) Find an equation of the line tangent to the path of the particle at time  $t = 0$ .
  - (c) Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 3$ .
  - (d) Find the  $x$ -coordinate of the position of the particle at time  $t = 3$ .
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2. Let  $f$  be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for  $f$  about  $x = 2$  is given by

$$T(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3.$$

- (a) Find  $f(2)$  and  $f''(2)$ .
  - (b) Is there enough information given to determine whether  $f$  has a critical point at  $x = 2$ ?  
If not, explain why not.  
If so, determine whether  $f(2)$  is a relative maximum, a relative minimum, or neither, and justify your answer.
  - (c) Use  $T(x)$  to find an approximation for  $f(0)$ . Is there enough information given to determine whether  $f$  has a critical point at  $x = 0$ ?  
If not, explain why not.  
If so, determine whether  $f(0)$  is a relative maximum, a relative minimum, or neither, and justify your answer.
  - (d) The fourth derivative of  $f$  satisfies the inequality  $|f^{(4)}(x)| \leq 6$  for all  $x$  in the closed interval  $[0, 2]$ . Use the Lagrange error bound on the approximation to  $f(0)$  found in part (c) to explain why  $f(0)$  is negative.
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**2004 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)**

$t$ (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity  $v(t)$ , in miles per minute at time  $t$  minutes, where  $v$  is a differentiable function of  $t$ . Selected values of  $v(t)$  for  $0 \leq t \leq 40$  are shown in the table above.
- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate  $\int_0^{40} v(t) dt$ . Show the computations that lead to your answer. Using correct units, explain the meaning of  $\int_0^{40} v(t) dt$  in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval  $0 < t < 40$ ? Justify your answer.
- (c) The function  $f$ , defined by  $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$ , is used to model the velocity of the plane, in miles per minute, for  $0 \leq t \leq 40$ . According to this model, what is the acceleration of the plane at  $t = 23$ ? Indicate units of measure.
- (d) According to the model  $f$ , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval  $0 \leq t \leq 40$ ?
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**END OF PART A OF SECTION II**

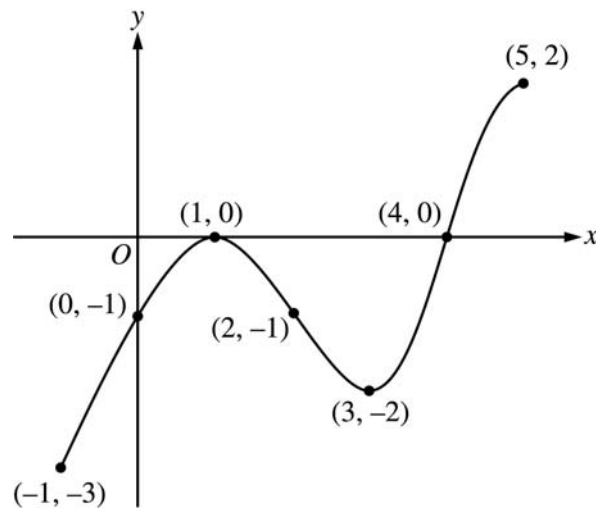
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CALCULUS BC  
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



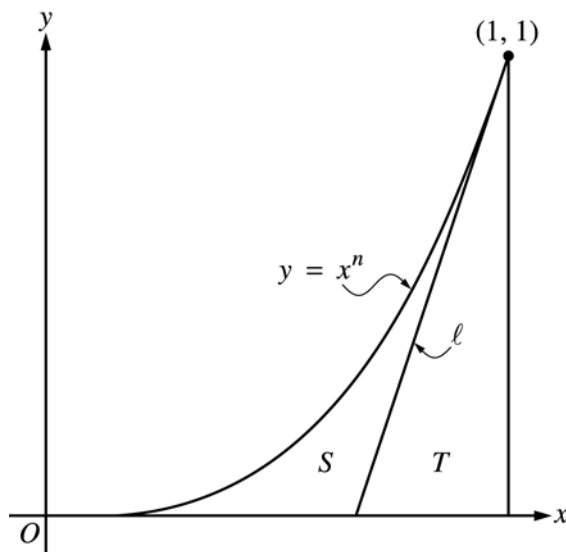
Graph of  $f'$

4. The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , on the closed interval  $-1 \leq x \leq 5$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$  and  $x = 3$ . The function  $f$  is twice differentiable with  $f(2) = 6$ .
- (a) Find the  $x$ -coordinate of each of the points of inflection of the graph of  $f$ . Give a reason for your answer.
- (b) At what value of  $x$  does  $f$  attain its absolute minimum value on the closed interval  $-1 \leq x \leq 5$ ? At what value of  $x$  does  $f$  attain its absolute maximum value on the closed interval  $-1 \leq x \leq 5$ ? Show the analysis that leads to your answers.
- (c) Let  $g$  be the function defined by  $g(x) = xf(x)$ . Find an equation for the line tangent to the graph of  $g$  at  $x = 2$ .

**2004 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)**

5. Let  $g$  be the function given by  $g(x) = \frac{1}{\sqrt{x}}$ .

- (a) Find the average value of  $g$  on the closed interval  $[1, 4]$ .
- (b) Let  $S$  be the solid generated when the region bounded by the graph of  $y = g(x)$ , the vertical lines  $x = 1$  and  $x = 4$ , and the  $x$ -axis is revolved about the  $x$ -axis. Find the volume of  $S$ .
- (c) For the solid  $S$ , given in part (b), find the average value of the areas of the cross sections perpendicular to the  $x$ -axis.
- (d) The average value of a function  $f$  on the unbounded interval  $[a, \infty)$  is defined to be  $\lim_{b \rightarrow \infty} \left[ \frac{\int_a^b f(x) dx}{b - a} \right]$ . Show that the improper integral  $\int_4^{\infty} g(x) dx$  is divergent, but the average value of  $g$  on the interval  $[4, \infty)$  is finite.



6. Let  $\ell$  be the line tangent to the graph of  $y = x^n$  at the point  $(1, 1)$ , where  $n > 1$ , as shown above.
- (a) Find  $\int_0^1 x^n dx$  in terms of  $n$ .
- (b) Let  $T$  be the triangular region bounded by  $\ell$ , the  $x$ -axis, and the line  $x = 1$ . Show that the area of  $T$  is  $\frac{1}{2n}$ .
- (c) Let  $S$  be the region bounded by the graph of  $y = x^n$ , the line  $\ell$ , and the  $x$ -axis. Express the area of  $S$  in terms of  $n$  and determine the value of  $n$  that maximizes the area of  $S$ .

**END OF EXAMINATION**